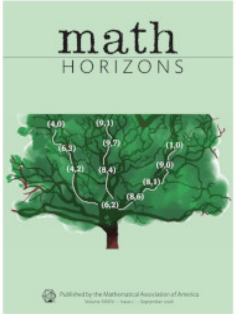

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A PERMUTATION ICEBREAKER, A TRUE EMBARKATION RECIPE

Egor Lappo, Maike L. Morrison, Noah A. Rosenberg, and Chloe E. Shiff

The scenario is familiar for many math students. You arrive at an organized activity among people who have not previously met, maybe a meeting of a student club, maybe a volunteering event—or any of innumerable other possible occasions. An organizer says, "Because we do not all know each other, let's do an icebreaker. We'll go around in a circle. Tell us your name, why you are here, and one interesting fact about yourself."

An "interesting fact," you wonder. What properties of a fact make it "interesting"? Do I *have* any "interesting facts"? The problem is unconstrained and underdetermined. Must the "interesting fact" be personal? Is anyone skipping the "interesting fact"? Do I really want to participate in this group at all?

Meanwhile, others effortlessly introduce themselves. Now it is your turn, and all eyes are on you. You say, "My interesting fact is that I hate icebreakers."

Icebreakers

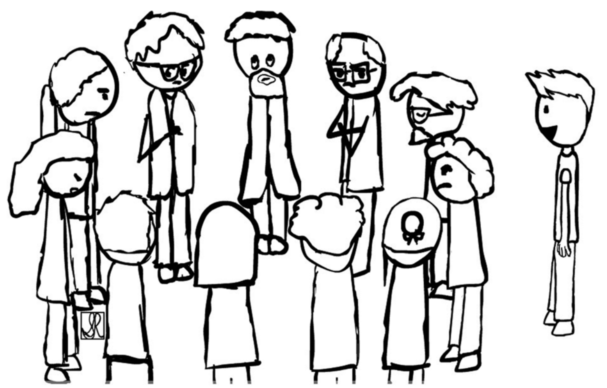
In this note, we describe an icebreaker that you can conduct to introduce the members of a group to one another without any "interesting facts"—and instead, with interesting mathematics.

To be precise, an *icebreaker* is an activity conducted in a group of people with the purpose of introducing its members, often in preparation for the group's effort to embark on a task together. Typically, participants are asked to share some fact about themselves in response to a prompt. The participants proceed in some sequence, specifying their name and personal fact.

Unease of participants with specifying personal facts is counterproductive to advancing the group's task. How can a group benefit from the camaraderie formed through an icebreaker while avoiding the dreaded personal facts? We have the following challenge:

> Devise an icebreaker activity in which no one is asked to share a personal fact.

Figure 1. Icebreakers. Drawing by Sonia Rosenberg.



"Everyone, please introduce yourself and share something you'd rather do than participate in an icebreaker."

The problem appears puzzling: Are personal facts not fundamental to an icebreaker? A solution lies in uncovering mathematical aspects of the activity itself.

We describe one approach that we recently implemented in a group of 38 people at an event on mathematical models for population biology at the Banff International Research Station.

An Icebreaker Recipe

As the organizer, a critical component of your task is to collect advance knowledge of the names of the participants. For each participant, obtain an anagram of the person's full name in advance and write it on an index card, ensuring a correspondence between n attending participants and their n anagrammed names.

Next, shuffle the cards, and to each of the n people, distribute a card selected at random. Speaking to the entire group, explain that participants are to unscramble the anagrams on their cards and search for the person whose name has been anagrammed. Specify that after several minutes, the group will reassemble, and beginning with a random person, each participant will share the anagram on the card and introduce the person whose name it permutes. To make the unscrambling easier, while people are searching, show a slide listing the names of all participants. The group will enjoy hearing the anagrams, and although each person has probability $1/n$ of a self-introduction, no one must share a personal fact.

Note that if the set of names has at least one pair that are anagrams of one another, then the map of participants to anagrams is not one-to-one. You can attempt to solve this problem with nicknames or middle initials. Alternatively, tell the group that anagrams might not uniquely specify a single person, and everyone should stop after finding one person whose permuted name matches the assigned card. In a group of typical size for icebreakers (say, 5–50), this ambiguity is unlikely.

Cycles in Random Permutations

The activity asks each participant to solve a small puzzle: to unscramble an anagram. Collectively, the group solves a puzzle as well, revealing the random permutation of n elements induced by passing out the cards.

As the activity unfolds, the random permutation is likely to contain loops that do not include all n people. In other words, if person A_1 introduces person A_2, A_2 introduces A_3, and so on, until person A_j for some $j < n$ introduces A_1, a loop that does not include all n people is completed—a *cycle*. After each cycle, the introductions must be restarted with a person who has not been included in the previous cycles.

For example, suppose the introductions follow table 1. Ailene (A_1) begins by introducing Brandon (A_2), who introduces Carl (A_3), who introduces Daniel (A_4), who introduces Emilia (A_5), who introduces Ailene, completing a cycle of length five. The table shows two cycles of length five and one cycle of length seven. Permutations can be written in *cycle notation*. For the example in table 1, we write the permutation of 17 people as a product of three cycles: (Ailene, Brandon, Carl, Daniel, Emilia) (Joanna, Kaleda, Lindi, Maria, Nicole, Oana, Puneeth) (Rohan, Sally, Troy, Ulrich, Viggo). In this notation, the order of elements inside a pair of parentheses represents a cycle, and the order in which the three cycles appear is not important.

What cycle lengths might we see among n people? The probability distribution of the cycle lengths in a random permutation follows a formula of Cauchy, described in the book *Logarithmic Combinatorial Structures* by Arratia, Barbour, and Tavaré. For a permutation selected at random from the $n!$ permutations of n elements, suppose that $D_j(n)$ gives the number of cycles of length j. We compute the probability distribution of $(D_1(n), D_2(n), \ldots, D_n(n))$: the probability that $(D_1(n), D_2(n), \ldots, D_n(n))$ equals $(d_1, d_2, \ldots, d_n)$ for an n-tuple of nonnegative integers d_j with $\sum_{j=1}^{n} j d_j = n$. In other words, we compute the probability that a random permutation of n elements has d_1 cycles of length 1, d_2 cycles of length 2, …, d_n cycles of length n. Cauchy's Theorem states:

$$\mathbb{P}[D_1(n) = d_1, D_2(n) = d_2, \ldots, D_n(n) = d_n] = \prod_{j=1}^{n} \left(\frac{1}{j}\right)^{d_j} \frac{1}{d_j!}.$$

In table 1, $(d_1, d_2, \ldots, d_{17}) = (0,0,0,0,2,0,1,0,\ldots,0)$. The probability that a random permutation of 17 elements has two cycles of length five ($d_5 = 2$) and one of length seven ($d_7 = 1$) is

$$\left[\left(\frac{1}{5}\right)^2 \frac{1}{2!}\right]\left[\left(\frac{1}{7}\right)^1 \frac{1}{1!}\right] = \frac{1}{350}.$$

To prove Cauchy's formula, observe that a permutation can be written (using cycle notation)

$$(X_1)(X_2)\ldots(X_{d_1})(X_{d_1+1}X_{d_1+2})(X_{d_1+3}X_{d_1+4})\ldots$$
$$(X_{d_1+2d_2-1}X_{d_1+2d_2})(X_{d_1+2d_2+1}X_{d_1+2d_2+2}X_{d_1+2d_2+3})\ldots,$$

where the X_k correspond to the elements A_k, permuted in some order. With table 1, the A_k would be the 17 people's names, Ailene, Brandon, …,

Table 1. An example permutation of n = 17 people.

Introducer	Person Who Is Introduced
Ailene	Brandon
Brandon	Carl
Carl	Daniel
Daniel	Emilia
Emilia	Ailene
Joanna	Kaleda
Kaleda	Lindi
Lindi	Maria
Maria	Nicole
Nicole	Oana
Oana	Puneeth
Puneeth	Joanna
Rohan	Sally
Sally	Troy
Troy	Ulrich
Ulrich	Viggo
Viggo	Rohan

Viggo, and each X_k would be chosen as one of the A_k, without replacement. The number of ways to assign the elements A_k to the variables X_k is $n!$. However, each distinct permutation of the A_k is obtained $\prod_{j=1}^{n} d_j!j^{d_j}$ times: Each set of d_j cycles of length j is obtained in each of $d_j!$ assignments of the A_k to the X_k, and each of the d_j cycles of length j is obtained by j (cyclic) permutations of its elements. Hence, if one of the $n!$ permutations is selected at random, the probability that counts $(d_1, d_2, \ldots, d_n)$ are obtained for its cycle lengths is

$$\frac{1}{n!}\frac{n!}{\prod_{j=1}^{n} d_j!j^{d_j}}.$$

Mean Number of Cycles

How many cycles are expected among the n people? Write $D(n) = D_1(n) + D_2(n) + \ldots + D_n(n)$ for the total number of cycles. We calculate the mean, or expected value, $\mathbb{E}[D(n)] = \mathbb{E}[D_1(n)] + \mathbb{E}[D_2(n)] + \ldots + \mathbb{E}[D_n(n)]$. Consider some specific cycle Z of length j. If Z appears in a permutation, then the remaining $n - j$ elements can be permuted in $(n - j)!$ ways. Hence, the probability that Z appears is $(n - j)!/n!$. The total number of distinct cycles of length j possible is

$$\frac{\binom{n}{j} j!}{j},$$

where the term $\binom{n}{j}$ represents the number of ways to choose j elements from the collection of n, the j elements can be ordered in $j!$ ways, and we divide by j to account for the fact that among these orderings, each possible cycle is produced j times (with the elements permuted cyclically). Each possible cycle of length j is equiprobable, so that to obtain $\mathbb{E}[D_j(n)]$, we multiply the probability of cycle Z by the number of possible cycles of length j, or

$$\mathbb{E}[D_j(n)] = \frac{(n-j)!}{n!}\binom{n}{j}\frac{j!}{j} = \frac{1}{j}.$$

Summing over all j, the mean number of cycles in a random permutation satisfies

$$\mathbb{E}[D(n)] = 1 + \frac{1}{2} + \ldots + \frac{1}{n}.$$

Permutations and Population Genetics

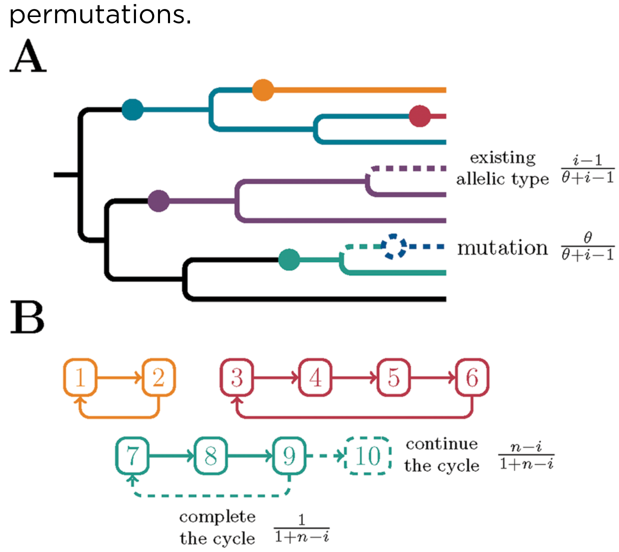
Random permutations are connected to a touchstone of mathematical population biology, the *Ewens Sampling Formula* (ESF). In genetics, for a location or *locus* in the genome where individuals vary in their genetic material, the distinct types that can be found are known as *alleles* or *allelic types*. The ESF describes the probability distribution of the counts of allelic types in a random sample from a large population, under a standard population-genetic model in which no allelic type is favored by natural selection.

In the model, individual genomes, each of which is measured at a locus, are connected via descent from a shared ancestor along the branches of a bifurcating genealogical tree. In figure 2A, a genealogical tree is shown for a set of genomes, moving forward in time from left to right. The *sample* of genomes whose allelic types are measured represents a collection of genetic lineages that trace to the shared ancestor (in whom, looking backward in time, the lineages "coalesce"). Mutations occur along the tree according to a rate parameter θ, often known as the *population-scaled mutation rate* (described further in Chapter 4 of the book *Coalescent Theory: An Introduction* by Wakeley). Each mutation, represented in figure 2A with a circle, generates a new allelic type, changing the color for the line of descent in the figure.

Suppose that $C_j(n)$, $j = 1, 2, \ldots, n$, describes the number of distinct allelic types represented j times in a random sample of size n. For any sample, $\sum_{j=1}^{n} jC_j(n) = n$. The ESF states that for an n-tuple of nonnegative integers $(c_1, c_2, \ldots, c_n)$ with $\sum_{j=1}^{n} jc_j = n$, the probability distribution of $(C_1(n), C_2(n), \ldots, C_n(n))$ satisfies

$$\mathbb{P}[C_1(n) = c_1, C_2(n) = c_2, \ldots, C_n(n) = c_n] = \frac{n!}{\theta(\theta+1)\cdots(\theta+n-1)}\prod_{j=1}^{n}\left(\frac{\theta}{j}\right)^{c_j}\frac{1}{c_j!}.$$

Figure 2. Linking (A) allelic sampling and (B) random permutations.



For example, if $n = 17$ genomes are examined, then the probability that three allelic types are observed in the sample, two seen five times ($c_5 = 2$) and one seen seven times ($c_7 = 1$), is obtained by using the ESF with $(c_1, c_2, \dots, c_{17}) = (0,0,0,0,2,0,1,0,\dots,0)$.

The ESF has an associated result for $\mathbb{E}[C(n)]$, the mean number of allelic types:

$$\mathbb{E}[C(n)] = \theta\left(\frac{1}{\theta} + \frac{1}{\theta+1} + \dots + \frac{1}{\theta+n-1}\right).$$

ESF and Random Permutations

The ESF, providing a probability distribution on partitions of n, appears in diverse problems in combinatorics and probability. In particular, it connects to problems on random permutations, for which the joint distribution of the cycle counts follows the ESF with $\theta = 1$.

What links the ESF to random permutations? We can sketch the connection through the lens of sequential processes. In the sampling that generates the ESF, conditional on $i-1$ draws from a population having already been obtained, the next draw adds a new lineage to the genealogical tree (figure 2A). The new lineage belongs to a new allelic type—a mutation—with probability $\theta / (\theta + i - 1)$, and to an existing type with probability $(i-1)/(\theta + i - 1)$. In the draw of a permutation uniformly at random during the icebreaker, conditional on $i-1$ introductions having already been made, the ith introduction either extends the current cycle or completes the cycle (figure 2B). It completes a cycle with probability $1/(1+n-i)$, as $1+n-i$ people remain to be introduced when the ith introduction takes place, and each is equally likely to be next; the ith introduction continues a cycle with probability $(n-i)/(1+n-i)$.

Across all n steps, the random permutation process can be shown to be equivalent to a special case of the allelic sampling process, the ESF itself corresponding to sampling permutations with a parameter that describes the extent to which the next element is biased toward ending or continuing the cycle.

Permutation-Based Games

The proposed icebreaker is only one among many whimsical scenarios that the theory of random permutations, and by extension, the ESF, helps to analyze. An article by Tavaré in the *Bulletin of the London Mathematical Society* collected several examples, including:

- The hat check problem: n people leave their hats at a hat check. Later, each person is returned one of the hats at random. What is the probability that no one receives the correct hat?
- The screaming toes problem: n people standing in a circle each look at someone else's feet. At a signal, everyone looks to the eyes of the corresponding person. If two people see each other, they scream. What is the probability that at least one pair screams?

In the icebreaker, the hat check problem seeks the probability that no one introduces themselves. The screaming toes problem seeks the probability of at least one pairwise introduction, conditional on no self-introductions.

Our Implementation

At our event in Banff, everyone unscrambled their assigned anagram (including HEAR NO MATH for our colleague ROHAN MEHTA). The cycles had lengths 19, 6, 5, 5, and 3; the number of cycles, 5, was near the mean for group size 38, or $1 + \frac{1}{2} + \dots \frac{1}{38} \approx 4.23$. No self-introductions occurred, as in the hat check problem. No one screamed.

At the conclusion of the activity, we shared the connection between the permutation-based icebreaker and the ESF—which, as a well-known topic in mathematical population biology, was familiar to the assembled participants. The problem of devising an icebreaker without the need to specify interesting facts has at least one solution, but we imagine that it is by no means unique. We invite readers to find other solutions.

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